A multidimensional method that eliminates internal multiples: a new tool box option for removing multiples that interfere with primaries, without damaging the primary, and without any knowledge of subsurface properties

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SUMMARY

Multiple removal is a longstanding problem in exploration seismology. Many methods have been developed including: stacking, FK filter, Radon transform, deconvolution and Feedback loop. They make statistical assumptions, assume move-out differences, or require knowledge of the subsurface and the generator of the multiples. As the industry went to deep water and more complex on-shore and off-shore geologic plays, these methods bumped up against their assumptions. The Inverse Scattering Series (ISS) internal-multiple-attenuation algorithm (Araújo et al. (1994),Weglein et al. (1997) and Weglein et al. (2003)) has stand-alone capabilities since it makes none of the assumptions of previous methods (listed above) and is especially effective when the subsurface is complicated and unknown. It is the only multi-dimensional internal-multiple-removal method that can predict all internal multiples with exact arrival time and approximate amplitude without requiring any subsurface information. When internal multiples and primaries are isolated, the ISS internal-multiple-attenuation algorithm is usually combined with an energy-minimization adaptive subtraction to remove internal multiples. For this case (isolated multiples), the ISS internal-multiple attenuator combined with energy-minimization adaptive subtraction is successful and effective. When internal multiples are proximal to and/or interfering with primaries or other events, the criteria of energy-minimization adaptive subtraction can fail (e.g., the energy can increase rather than decrease when a multiple is removed from a destructively interfering primary and multiple). With interfering events, this failure of energy-minimization adaptive subtraction can lead to removing/damaging the target primary which is the worst possible outcome. In this paper, we provide the first multi-dimensional ISS internal-multiple-elimination algorithm that can predict both the correct time and amplitude of internal multiples. This is an important part of a three-pronged strategy proposed by Weglein at the 2013 SEG International Conference (Weglein 2014). This three-pronged strategy provides an effective response to this challenge (interfering primaries and internal multiples) while retaining and adding to the strengths of the current ISS internal-multiple-attenuation algorithm. Herrera and Weglein (2012) proposed a 1D internal-multiple-elimination algorithm for all first-order internal-multiples generated at the shallowest reflector. Y. Zou and Weglein (2014) developed and illustrated an elimination algorithm that can eliminate all first-order internal multiples generated by all reflectors for a 1D earth. In this paper we provide the first multidimensional ISS internal-multiple-elimination method that can remove internal multiples interfering with primaries, without subsurface information, and without damaging the primary. We also compare the ISS elimination result with attenuation plus energy-minimization adaptive subtraction for an interfering primary and internal multiple. This ISS internal-multiple-elimination algorithm is more effective and more compute-intensive than the current most capable ISS attenuation-plus-adaptive-subtraction method. We provide it as a new capability in the multiple-removal toolbox and a new option for circumstances when this type of capability is called for, indicated and necessary.

INTRODUCTION

The ISS (Inverse-Scattering-Series) allows all seismic processing objectives, e.g., free-surface-multiple removal and internal-multiple removal to be achieved directly in terms of data, without any need for, or estimation of, the earth's properties. The ISS internal-multiple attenuation algorithm is the only method today that can predict the correct time and approximate and well-understood amplitude for all first-order internal multiples generated from all reflectors, at once, without any subsurface information. If the multiple to be removed is isolated from other events, then the energy minimization adaptive subtraction can fill the gap between the attenuation algorithm amplitude prediction and the amplitude of the internal multiples. However primary and multiple events can often interfere with each other in both on-shore and off-shore seismic data. In these cases, the criteria of energy minimization adaptive subtraction can fail and completely removing internal multiples becomes: (1) more challenging and (2) beyond the current collective capability of the petroleum industry.

For dealing with this challenging problem, Weglein (2014) proposed a three-pronged strategy:

1. Develop the ISS prerequisites for on-shore applications (e.g. removing and predicting the reference wave field and to produce de-ghosted data).
2. Develop internal-multiple elimination algorithms from ISS.
3. Develop a replacement for the energy-minimization criteria for adaptive subtraction that derives from and always aligns with the ISS elimination algorithm.

To achieve the second part of the strategy, that is, to upgrade the ISS internal-multiple attenuation algorithm to elimination algorithm, the strengths and limitations of the ISS internal-multiple attenuation algorithm are noted and reviewed. The ISS internal-multiple attenuation algorithm always attenuates all internal multiples from all reflectors at once, automatically and without any subsurface information. That unique strength is always present and is independent of the circumstances and complexity of the geology and the play. However, there are two well-understood limitations of this ISS internal-multiple attenuation algorithm

1. It may generate spurious events due to internal multiples treated as sub-events.
2. It is an attenuation algorithm not an elimination algorithm.

The first item is a shortcoming of the leading order term (the term used to derive the current attenuation algorithm), when taken in isolation, **but is not an issue for the entire ISS internal-multiple removal capability**. It is anticipated by the ISS and higher order ISS internal multiple terms exist to precisely remove that issue of spurious events prediction. When taken together with the higher order terms, the ISS internal multiple removal algorithm no longer experiences spurious events prediction. Ma et al. (2012) , H. Liang and Weglein (2012) and Ma and Weglein (2014) provided those higher order terms for spurious events removal.

In a similar way, there are higher order ISS internal multiple terms that provide the elimination of internal multiples when taken together with the leading order attenuation term. The initial idea is provided by Weglein and Matson (1998) in which the attenuation factor, which is a collection of extra transmission coefficients and is the difference between attenuation and elimination, is systematically studied. Later there are furthur discussions in Ramírez (2007). Several extensions are proposed based on the initial idea. Herrera and Weglein (2012) proposed an algorithm for internal multiple elimination for all first order internal multiples generated at the first reflector. Benefited from the previous work, Zou and Weglein (2014) proposed an new algorithm that can eliminate all first order internal multiples for all reflectors for a 1D earth. In this paper, we further extend the previous elimination algorithm and provide the first ISS multi-dimensional elimination method for all first order internal multiples. The new elimination algorithm retains the benefits of the attenuation algorithm, including not requiring any subsurface information and unlike stripping methods, removes all first-order internal multiples from all subsurface reflectors at once.

THE ISS INTERNAL-MULTIPLE ATTENUATION ALGORITHM AND THE INITIAL IDEA FOR INTERNAL MULTIPLE ELIMINATION

The ISS internal-multiple attenuation algorithm is first given by Araújo et al. (1994) and Weglein et al. (1997). The 1D normal incidence version of the algorithm is presented as follows (The 2D version is given in Araújo et al. (1994),Weglein et al. (1997) and Weglein et al. (2003) and the 3D version is a straightforward extension.),

$$b_3(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} b_1(z') \times \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z''). \tag{1}$$

Where $b_1(z)$ is the constant velocity Stolt migration of the data of a 1D normal incidence spike plane wave. ε_1 and ε_2 are two small positive numbers introduced to avoid self interactions. $b_3^{IM}(k)$ is the predicted internal multiples in the vertical wavenumber domain. This algorithm can predict the correct time and approximate amplitude of all first-order internal multiples at once without any subsurface information.

The ISS internal-multiple attenuation algorithm automatically uses three primaries in the data to predict a first-order internal multiple. (Note that this algorithm is model type independent and it takes account all possible combinations of primaries that can predict internal multiples.). Every sub event experiences several phenomena making its way down to the earth then back to the receiver. When compared with the actual internal multiple, the prediction has extra transmission coefficients. Multiplying all those extra transmission coefficients, we get the AF (attenuation factor) - $T_{01}T_{10}$ for this first-order internal multiple generated at the shallowest reflector. And all first-order internal multiples generated at the shallowest reflector have the same attenuation factor.

The attenuation factor for predicting a multiple generated by the i^{th} reflector, AF_j, is given by the following:

$$AF_j = \begin{cases} T_{0,1}T_{1,0} \quad (for\ j=1) \\ \prod_{i=1}^{j-1}(T_{i-1,i}^2 T_{i,i-1}^2)T_{j,j-1}T_{j-1,j} \quad (for\ 1<j<J) \end{cases} \tag{2}$$

The subscript j represents the generating reflector, and J is the total number of interfaces in the model. The interfaces are numbered starting with the shallowest location. The attenuation factor is a collection of extra transmission coefficients and is the difference between attenuation and elimination. Weglein and Matson (1998) studied the attenuation factor and provide the initial idea and algorithm to remove the attenuation factor by reflection data to achieve the elimination.

As discussed in Weglein and Matson (1998), the attenuation algorithm prediction contains the attenuation factor and in order to develop an elimination algorithm, we should remove the attenuation factor. However, the attenuation factor is expressed using transmission coefficients. Since the data contains reflection coefficients, the idea is to use reflection coefficients to represent the transmission coefficient such that we can remove the attenuation factor by the data (which contains reflection coefficients) without any subsurface information. For example, to remove AF_1 in the prediction, we have

$$elimination = \frac{attenuation}{AF_1} = \frac{attenuation}{T_{0,1}T_{1,0}} = \frac{attenuation}{1-R_1^2} \tag{3}$$

$$= attenuation + attenuation \times R_1^2 + attenuation \times R_1^4 + ...$$

where the first term is the attenuation algorithm, the term $attenuation \times R_1^2$ corresponds to the first higher order term towards elimination and so on. A detailed discussion can be found in Weglein and Matson (1998) and Ramírez (2007).

THE FIRST INVERSE-SCATTERING-SERIES INTERNAL MULTIPLE ELIMINATION METHOD FOR A MULTI-DIMENSIONAL SUBSURFACE

The Inverse-Scattering-Series contains an internal-multiple elimination sub-series. Since the internal multiple attenuation algorithm is capable to predict the correct time and approximate amplitude for all internal multiples, if we can isolate all terms that can predict the same time as the attenuation algorithm by the initial elimination idea (removing the attenuation factor by the reflection data) in the Inverse-Scattering-Series, then adding all these terms together will give us an elimination algorithm. Since the Inverse-Scattering-Series is a multi-D series, the elimination algorithm/terms identified is a multi-D algorithm. Benefited from the previous work, we propose a Inverse-Scattering-Series internal multiple elimination method that can eliminate all first-order internal multiples for all reflectors for a multi–dimensional subsurface. Below shows a 2D version of a higher order term in the elimination algorithm.

$$b_E(k_s,k_g,q_g+q_s) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dk_1 dk_2 \int_{-\infty}^{+\infty} dz_1 b_1(k_g,k_1,z_1) e^{i(q_g+q_1)z_1} \times \int_{-\infty}^{z_1-\varepsilon} dz_2 F(k_1,k_2,z_2) e^{-i(q_1+q_2)z_2} \int_{z_2+\varepsilon}^{+\infty} dz_3 b_1(k_2,k_s,z_3) e^{i(q_2+q_s)z_3} \tag{4}$$

$$F(k_1,k_2,z) = \int_{-\infty}^{+\infty} d(q_1+q_2) e^{-i(q_1+q_2)z} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dk' dk'' \int_{-\infty}^{+\infty} dz' b_1(k_1,k',z') e^{i(q_1+q')z'} \times \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(k',k'',z'') e^{-i(q'+q'')z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g(k'',k_2,z''') e^{i(q''+q_2)z'''} \tag{5}$$

$$g(k_1,k_2,z) = \int_{-\infty}^{+\infty} d(q_1+q_2) e^{-i(q_1+q_2)z} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dk' dk'' \int_{-\infty}^{+\infty} dz' b_1(k_1,k',z') e^{i(q_1+q')z'} \times \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' b_1(k',k'',z'') e^{-i(q'+q'')z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g(k'',k_2,z''') e^{i(q''+q_2)z'''} \tag{6}$$

Similar to the extension in the previous work in Zou and Weglein (2014) , $F(k_1,k_2,z)$ and $g(k_1,k_2,z)$ are two intermediate functions. Combining all of these kind of higher order terms provides the elimination algorithm in a 2D earth. The elimination algorithm for a 3D earth is a straightforward extension.The complete subseries is given in Zou et al. (2016).

SYNTHETIC DATA EXAMPLE

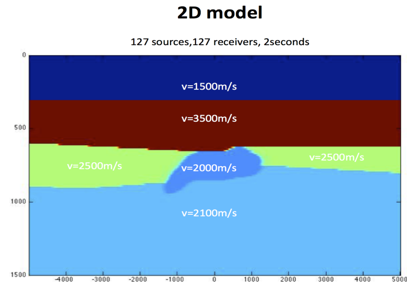

Figure 1: 2D Model

Figure 1 shows the 2D model, the data is generated by finite difference method. There is a hat shape structure in the middle of the model. The shape is designed so that its lower boundary is negatively interfering with an internal multiple. In this synthetic data, there are 127 shot gathers, each shot gather contains 127 receivers. The source and receiver interval is 30 meters and time interval 0.002s.

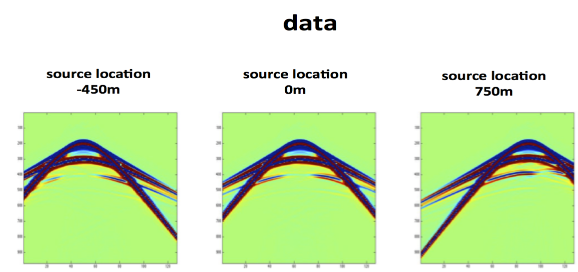

Figure 2: pre-stack data

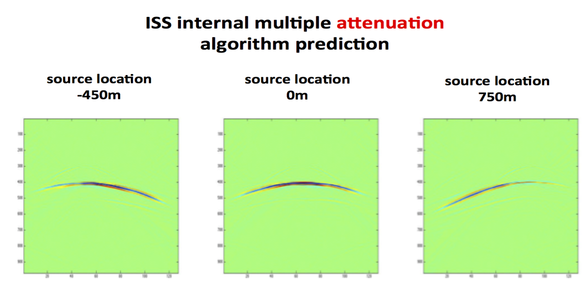

Figure 3: ISS internal multiple attenuation algorithm prediction

Figure 2 shows three shot gathers of the input data with source at -450m, 0m, 750m respectively. Figure 3 shows three shot gathers of the ISS attenuation algorithm prediction with source at -450m, 0m, 750m respectively. Figure 4 shows three shot gathers of the ISS elimination algorithm prediction with source at -450m, 0m, 750m respectively. From these figures we can see the strongest internal multiple is interfere with one primary in the data, especially at small offset.

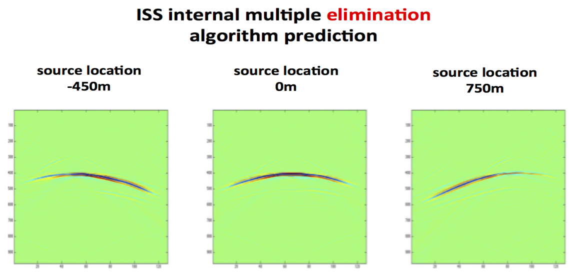

Figure 4: ISS internal multiple elimination algorithm prediction

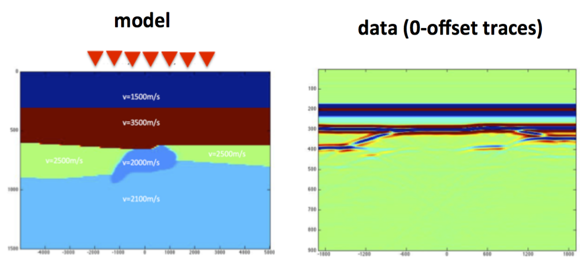

Figure 5: Zero offset traces of data

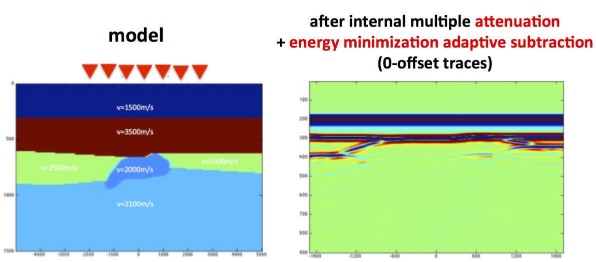

Figure 6: Zero offset traces after ISS internal multiple attenuation and energy minimization adaptive subtraction

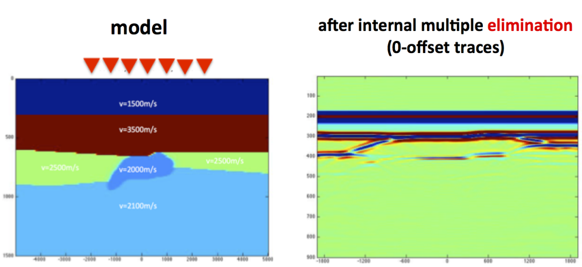

Figure 7: Zero offset traces after ISS internal multiple elimination

In order to see the result more clearly, we show the zero offset traces results. Figure 5 shows the zero offset traces of the input data. Compared to the model, we can see clearly that the lower boundary of the hat shape is almost invisible because the primary generated by the hat's lower boundary is negatively interfere with an internal multiple. Figure 6 shows the zero offset traces results after ISS internal multiple attenuation and adaptive subtraction. We can see the lower boundary of the hat shape is still not visible. It is because the criteria of the energy minimization adaptive subtraction fails, that is, the primary energy after subtraction is larger than the interfering events. Figure 7 shows the 0-offset trace results after internal multiple elimination. The lower boundary of the hat shape is recovered in the result. It proves that the elimination algorithm can predict both correct time and amplitude and can eliminate internal multiples without touching the primary.

CONCLUSION

The ISS multi-dimensional internal-multiple-elimination algorithm that removes internal multiples (while accommodating primaries and internal multiples as sub-events) is one part of the three-pronged strategy that is a direct response to current seismic processing and interpretation challenge when primaries and internal multiples are proximal to and/or interfere with each other in both on-shore and off-shore plays. This new algorithm addresses the shortcomings of the current most capable internal-multiple-removal method (ISS internal-multiple-attenuation algorithm plus adaptive subtraction). Meanwhile, this elimination algorithm retains the stand-alone benefits of the ISS internal-multiple-attenuation algorithm that can predict all internal multiples at once and requiring no subsurface information (in contrast to stripping and Feedback loop methods that remove multiples layer by layer and require subsurface information). This ISS internal-multiple-elimination algorithm is more effective and more compute-intensive than the current best internal-multiple-removal method. Within the three-pronged strategy, our plans include developing an alternative adaptive-subtraction criteria for internal-multiple elimination derived from, and always aligned with the ISS elimination algorithm. That would be analogous to the new adaptive criteria for free-surface-multiple removal proposed by Weglein (2012), as a replacement for the energy-minimization criteria for adaptive subtraction. We provide this new multi-dimensional internal-multiple-elimination method as a new internal-multiple-removal capability in the multiple-removal toolbox that can remove internal multiples that interfere with primaries without subsurface information and without damaging the primary.

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